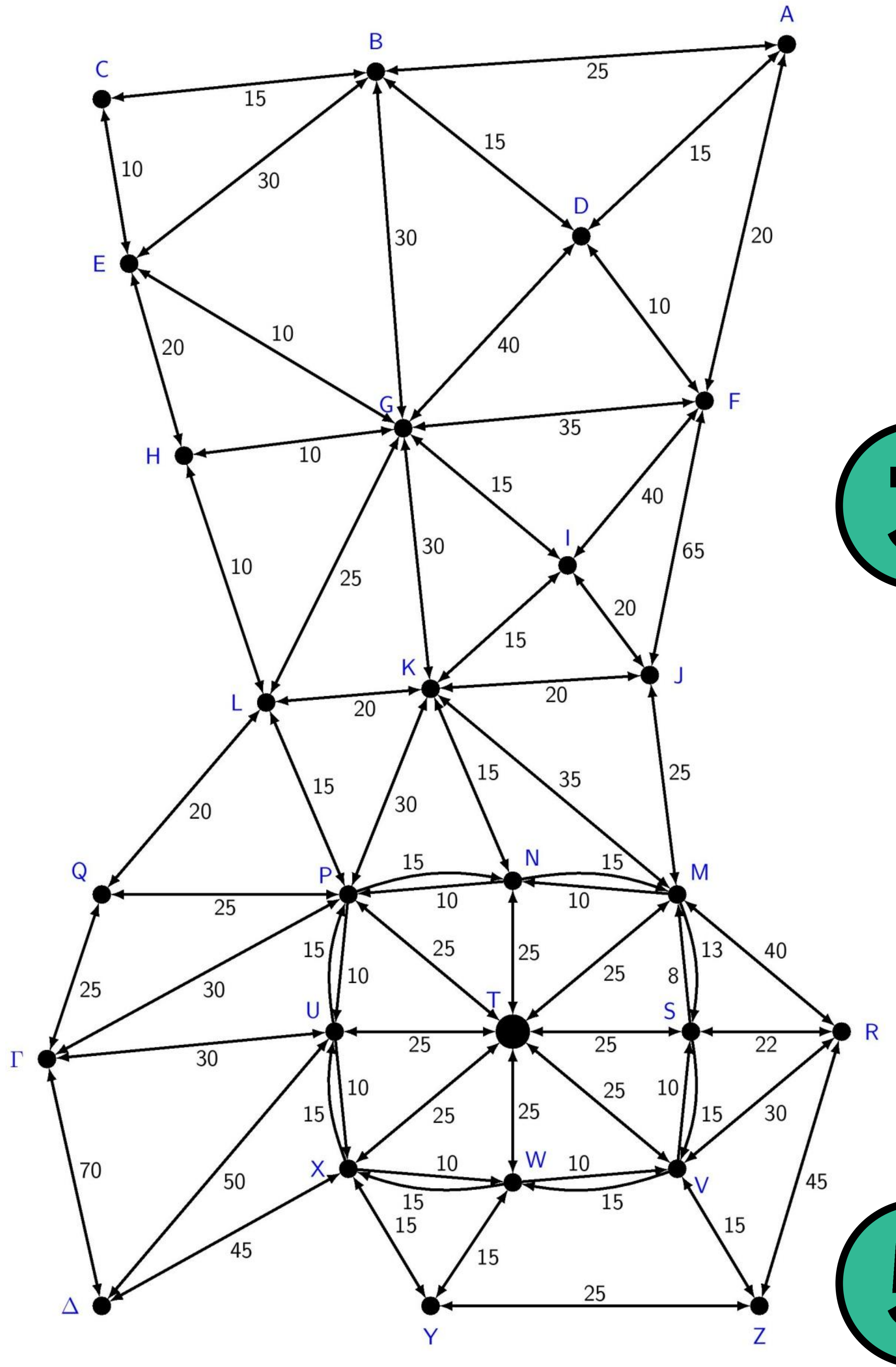
Graph Theory: How Google Maps Plans Routes

The Problem

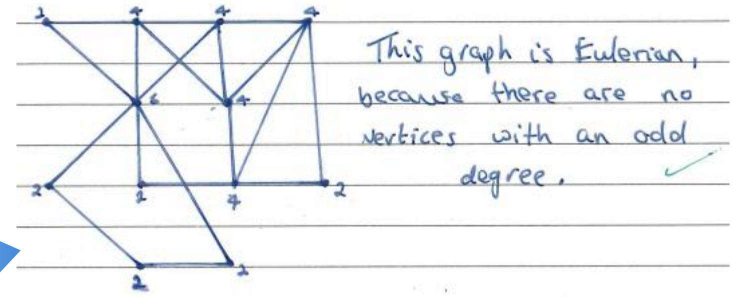
As part of their Final Assignment, pupils had to calculate the quickest route from point A to point Z in this graph – a model of a collection of cities linked by roads. Throughout the course, they studied graph theory, equipping them with the knowledge and skills to solve this problem.



Tutorial 1: What is a graph?

Pupils began by learning that a graph is a collection of points (vertices) linked by lines (edges). They discovered Leonhard Euler invented graphs to solve the Seven Bridges of Konigsberg Problem. We also learned that graphs can be used to model computer networks and transport systems. Graphs are simple structures, and rudimentary mathematics such as parity properties of integers can be used to prove unexpected facts: only a graph in which all its vertices are of even degree can be drawn, starting anywhere, without retracing one's steps. Such graphs are called Eulerian.

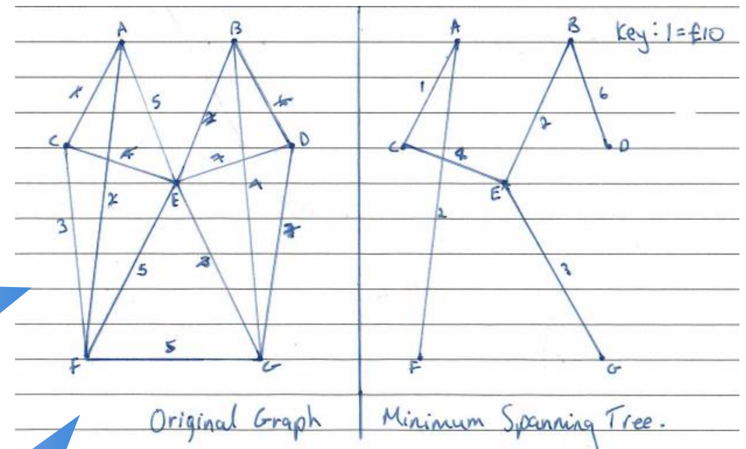
As part of their homework, pupils researched weighted graphs and how they can be applied to real world situations – this is an essential concept later in the course.



Tutorial 2: Cycles and Trees

Here, we studied an important substructure of graphs: cycles. A cycle is a journey along edges from vertex to vertex that ends where it began. A connected graph with no cycles is called a tree. With the application of more basic mathematics, pupils were able to discover that every tree with n vertices contains n-1 edges.

Pupils researched algorithms as part of their homework, which leads into the next tutorial.



Tutorial 3: Kruskal's Algorithm

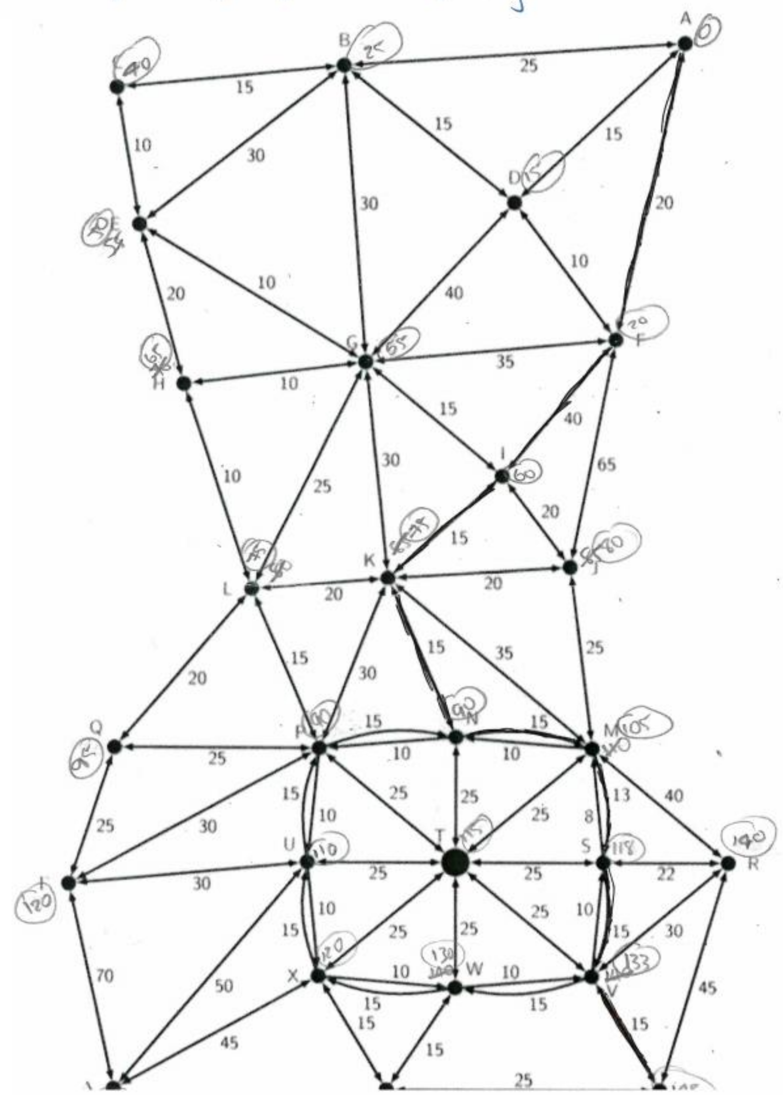
Pupils learned about minimum spanning trees – the "skeleton" of a graph, and their importance in the practical applications of graphs. Minimum spanning trees give the cheapest way of connecting cities or computers in a network, but we also explored drawbacks like potential increased travel times and decreased network stability. We then learned Kruskal's Algorithm, a procedure for finding the minimum spanning tree of a graph.

In this week's research question, pupils learned about walks, trails, and paths. These are the various ways in which graphs can be traversed, and is a key component in the next tutorial, where we address the main problem of the course.

Tutorial 4: Dijkstra's Algorithm

We learned Dijkstra's Algorithm, a method for finding the quickest route from one point to another on a graph. This method forms the basis for the more sophisticated algorithms found in GPS systems and Google Maps.

In their homework, pupils researched directed graphs and wrote about why we might use them over undirected graphs.



Tutorial 5: Consolidation

In the final tutorial, we applied Dijkstra's Algorithm in more complex scenarios, like those involving directed graphs or with varying traffic. Pupils were now equipped to tackle the main problem in their Final Assignment.

Final Assignment

Here we see that every tutorial has granted this pupil with the skills to produce an excellent Final Assignment. Equipped with refined research skills, they also went on to independently learn and implement the Route Inspection algorithm.

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